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Pierre Pestieau* and Gregory Ponthiere†

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Abstract

Although the optimal policy under endogenous fertility has been widely studied, the optimal public intervention under endogenous childbearing age has remained largely unexplored. This paper examines the optimal family policy in a context where the number and the timing of births are chosen by individuals who differ as to how early fertility can weaken future earnings growth. We analyze the design of a policy of family allowances and of public pensions in such a setting, under distinct informational environments. We show how endogenous childbearing ages affect the optimal policy, through the redistribution across the earnings dimension and the internalization of fertility externalities. It is also shown that, contrary to common practice, children benefits differentiated according to the age of parents can, under some conditions, be part of the optimal family policy.

Keywords: endogenous fertility, childbearing age, pensions, family benefits.

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1 Introduction

As this is well-known among demographers, there has been a continuous postponement of fertility in European economies since the late 1970s. That strong demographic trend is well illustrated by the case of France. As shown on Figure 1, the average age at motherhood has raised from about 26.9 years in 1977 to about 29.7 years in 2005.[1] The postponement of fertility appears even more strongly when we look at the mode of the distribution of the age at motherhood: the mode age has grown from 25 years in 1977 to 29 years today. With the development of assisted reproductive technologies, that evolution is likely to be sustained - if not reinforced - in the future.

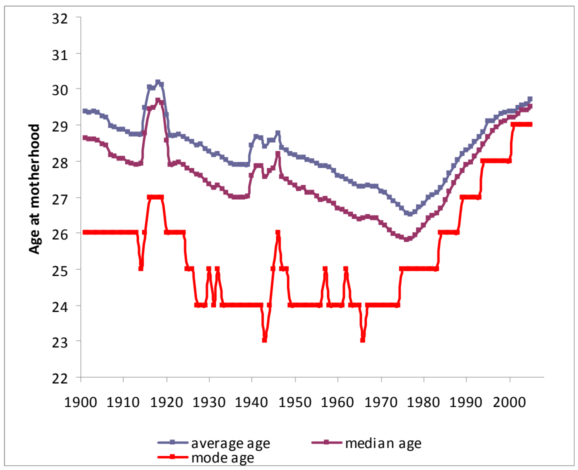

Figure 1: Age at motherhood in France

The causes of the postponement of fertility have been widely explored.[2] Empirical studies highlighted that women's earnings opportunities and educational achievements are a major determinant of fertility behavior. The rise in women's wage, by implying a higher opportunity cost of motherhood, appears to be a major factor of fertility decline, which coincides also with later motherhood.[3]

Besides those empirical studies, theoretical models have also been developed to explain the choice of a particular fertility age-pattern over the lifecycle. Those models, such as Happel *et al.* (1984), Cigno and Ermisch (1989) and Cigno (1991), highlighted the central influence of lifecycle effects on the tim-

[1] Sources: INED (2011).

[2] See Gustafsson (2001) for a survey on the literature on childbearing age.

[3] Empirical studies include: Schultz (1985), Heckman and Walker (1990) and Tasiran (1995) on Sweden, Ermisch and Ogawa (1994) on Japan, Merrigan and St Pierre (1998) on Canada, and Joshi (2002) on Great Britain.

ing of births. Whereas Happel *et al.* (1984) emphasized that the consumption smoothing induced by the maximization of lifetime welfare may lead to delaying births, Cigno and Ermisch (1989) focused on the shape of the earnings profile induced by human capital investment, and argued that steeper earnings profiles lead to the postponement of births later in life.[4]

The various influences of governments on the timing of births have also been studied. While the income tax rate and parental leave benefits reduce the opportunity cost of early children, children allowances reduce the net direct expenditures, and the income tax rate reduces also the forgone returns to forgone human capital investment. Those influences of governments have been confirmed by various empirical studies.[5] Macroeconometric studies, such as Ermisch (1988) on UK and Walker (1995) on Sweden, identified a positive impact of child allowances on early motherhood. That effect was confirmed by microeconometric studies, such as Laroque and Salanié (2004) on France, who found that cash benefits increase the probability of having a first child.

In the light of those influences of the government on fertility behavior, a natural question to raise concerns the optimal design of family policies. What should government do in front of the postponement of fertility? Should governments implement a transfer policy in such a way as to reinforce (or weaken) that demographic trend? Should governments subsidize births differently depending on the age of parents? By which channels are the optimal fertility policy and the optimal pension policy related to each other?

The literature on the optimal policy under endogenous fertility has examined various aspects of the problem, but without paying attention to the optimal *timing* of births. Cigno (1986) showed that family allowances aimed at reducing child poverty may, by raising fertility, have quite the opposite effects, so that the optimal policy may consist in taxing - instead of subsidizing - the number of children.[6] More recently, Cremer *et al.* (2006) studied the design of the optimal pension system under endogenous and stochastic fertility, and showed that, under positive fertility externalities, one should grant parents who have more children with larger pension benefits.[7]

The goal of this paper is to complement that literature, by examining the design of the optimal public intervention in a context where not only fertility, but, also, the childbearing age, are endogenous. For that purpose, we develop a three-period model. In the first two periods, individuals are active, and they can have children. In the third period, individuals retire. In order to account for the widely observed effect of heterogeneity in education and in the timing of births (Cigno and Ermisch 1989), we assume that some agents benefit from a rise in productivity and earnings (to an extent that is decreasing in the number

[4] Cigno and Ermisch (1989) also found, on the basis of UK data, empirical support for that explanation of the observed heterogeneity in terms of fertility patterns.

[5] On this, see the survey by Gauthier (2007).

[6] This was confirmed by Balestrino *et al.* (2002) in a model where households differ in their productivities and in their ability to raise children.

[7] That result is also obtained in Cremer *et al.* (2008), who examined the optimal pensions scheme when fertility is endogenous and parents differ in their ability to raise children.

of early children), whereas other agents do not benefit from such a process, and face a flat earnings profile. In that framework, having a child in the first period relative to the second is cheaper, but it may impose a cost in terms of educational and professional achievement, so that some trade-off exists between lower fertility cost and higher future wages. Moreover, when parents retire, and count on some help from their children, they will, if earnings increase with age, receive fewer resources per child under late childbearing than under early childbearing. Retired agents will also benefit from the contributions of fewer individuals under late childbearing than under early childbearing (where they benefit from the contributions of their working children *and* grandchildren).

In the following, we made several simplifying assumptions. First, we abstract here from intrafamily decision making issues, and treat a couple as a single individual capable of working, saving and having children. Second, we assume away the childhood period, i.e. the period during which children live as dependent of their parents. Third, we focus on a static overlapping-generations model, and do not explore here the effects of childbearing ages on the long-run dynamics.[8] Fourth, we assume here that the number of children is deterministic, and thus rule out uncertainty about the number of children.[9]

Anticipating our results, we show that, under the laissez-faire, agents who face flat earnings profile tend to make children earlier than agents with an increasing earnings profile. We characterize the first-best utilitarian social optimum, and show that this can be decentralized by means of lump sum transfers from increasing-productivity agents to constant-productivity agents, in order to equalize the consumption per period and the total number of children for all agents (despite different timings for births). Then, we consider the second-best optimal policy under linear taxation instruments, and show that the optimal family allowances are positive only if subsidizing children favours early childbearing. The study of the second-best problem under non-linear instruments shows that children allowances should concern children from constant-productivity parents only. Finally, we examine the optimal family policy under the presence of a Pay-As-You-Go (PAYG) pension system, and show that the social planner faces, under endogenous childbearing age, a trade-off between efficiency (favouring late childbearing) and equity (helping early childbearing).

The rest of the paper is organized as follows. Section 2 presents the general framework. The first-best utilitarian optimum is derived in Section 3, where its decentralization is also examined. In Section 4, we look at the optimal child benefits when there is a perfect capital market. In Section 5, we introduce the idea that pensions can be of the PAYG type, which may give an incentive to both the government and individuals to have early children. Section 6 concludes.

[8] Those effects are discussed in the companion paper Pestieau and Ponthiere (2011). It is shown there that, compared to the 2-period OLG model, the conditions for optimal capital accumulation and optimal fertility differ quite a lot under varying childbearing ages. However, Samuelson's (1975) Serendipity Theorem still holds in that broader demographic environment.

[9] See Cigno and Luporini (2009) on the optimal family policy when the number and the future earnings of children are random. The absence of risk rules out a case for early childbearing: the possibility to insure oneself against a total number of children lower than desired.

2 Basic model

2.1 Environment

We consider a three period model.[10] The periods are labeled a, ℓ and o (i.e. advanced, late, old). In the first two periods, individuals work, consume, save and have children. It is assumed that a perfect capital market exists. In the last period, individuals are retired and consume the proceeds of their savings.

To do justice to the observed heterogeneity in terms of education, career and childbearing ages, we assume that individuals differ in the shape of the earnings lifecycle profile. For simplicity, we consider the following two types:

- Type-1 agents have a constant productivity and earnings along the lifecycle (periods 1 and 2);[11]
- Type-2 agents have, at period 2, a productivity that is larger than in period 1, to an extent that is decreasing in the number of children they had in period 1.

The lifetime welfare of an individual of type $i \in \{1, 2\}$ is expressed as:

$$U_i = u(c_i) + \beta u(d_i) + \beta^2 u(b_i) + v(n_{ai} + n_{\ell i}) \tag{1}$$

where c_i, d_i and b_i are consumption, $u(\cdot)$ is strictly concave, β is a time preference factor ($0 \leq \beta \leq 1$) and $v(\cdot)$ is the utility for early and late children, which is also strictly concave.[12]

In the first period, consumption is equal to wage, w, minus saving and minus the cost of raising children, $n_{ai}e_a$:

$$c_i = w - s_{ai} - n_{ai}e_a. \tag{2}$$

In the second period, an individual of type $i \in \{1, 2\}$ earns $wh_i(n_{ai})$, with $h_2(n_{a2}) > h_1(n_{a1}) \geq 1$ and $h_i'(n_{ai}) < 0$. This reflects the idea that, for some individuals with a broad career potential, early childbearing has a cost on earning in period ℓ. Indeed, various empirical studies, such as Joshi (1990, 1998) on Great Britain and Dankmeyer (1996) on the Netherlands, showed that having children in an early stage of a career slows down human capital accumulation and professional promotion.[13]

[10] Actually, we consider a model that corresponds to the steady-state of an OLG model wherein the longitudinal view coincides with crossesectional one.

[11] In some cases below, we allow type-1 agents to have a higher and endogenous productivity in the second period, but always lower than that of type-2 agents.

[12] We assume here a perfect substituability, in welfare terms, between early and late children. That assumption, which is made for simplicity, amounts to assume that children are not durable consumption goods. As stressed in Gustafsson (2001), taking children as durable goods would favour early childbearing *ceteris paribus*.

[13] Note that the precise causes of the negative effect of early childbearing remain largely unknown. For instance, Ermisch and Pevalin (2005) showed that very early motherhood (teen births) worsens later outcomes on the marriage market.

In the second period, the individual earns also savings income Rs_{ai}, where R is the interest factor. He saves also $s_{\ell i}$ for the old age, and spends $e_\ell n_{\ell i}$ as cost of raising his $n_{\ell i}$ late children. Throughout this paper, we assume that the direct cost of children is larger for late children than for early children:

$$e_\ell > e_a$$

That assumption is compatible with the medical literature showing the larger costs of late motherhood in comparison to the ones under early motherhood. Those additional costs are of various kinds, and concern both the mother and the child (see Gilbert *et al.* 1999, Gustafsson 2001).[14] We assume, for simplicity, that the parents face the entire additional cost from late childbearing. Hence, second-period consumption for an agent of type $i \in \{1,2\}$ is:

$$d_i = wh_i(n_{ai}) + Rs_{ai} - s_{\ell i} - n_{\ell i}e_\ell. \tag{3}$$

In the third period, an agent of type $i \in \{1,2\}$ consumes the proceeds of his savings:

$$b_i = Rs_{\ell i}. \tag{4}$$

2.2 Laissez-faire

Let us characterize the laissez-faire equilibrium, where each agent of type $i \in \{1,2\}$ chooses first-, second- and third-period consumptions (i.e. c_i, d_i and b_i), as well as first- and second-period children (i.e. n_{ai} and $n_{\ell i}$) in such a way as to maximize his lifetime welfare, subject to his budget constraint.

For an agent of type $i \in \{1,2\}$, the first-order conditions are:

$$\begin{gathered} u'(c_i) = \beta R u'(d_i) = \beta^2 R u'(b_i) \\ -u'(c_i)e_a + \beta u'(d_i) w h_i'(n_{ai}) + v'(n_{ai} + n_{\ell i}) = 0 \\ -\beta u'(d_i)e_\ell + v'(n_{ai} + n_{\ell i}) = 0. \end{gathered}$$

The first equalities, which describe the optimal consumption profile, are quite standard. However, the second and third conditions, which characterize the optimal early and late number of children, are not straightforward.

To interpret those conditions, let us consider the case where $h_1(n_{a1}) = 1$ and assume further that $\beta = R = 1$. Then we have, at the laissez-faire equilibrium, a perfect consumption smoothing for all agents:

$$c_i = d_i = b_i$$

with higher consumption levels for type-2 agents, who benefit from an increasing earnings profile.

[14] According to Gustafsson (2001), late births imply, on average, more pregnancy complications, more ceasareans and more breast cancer for mothers. Moreover, late children are also more subject to somatic and learning problems.

We also have that type-1 agents have no children in the second period:

$$n_{\ell 1} = 0$$

Actually, given that late children are more costly than early children under $e_\ell > e_a$, and given that their productivity will remain the same whatever the number of early children is, type-1 agents have no reason to postpone fertility. The number of early children for type-1 agents is given by:

$$u'(c_1)\, e_a = v'(n_{a1})$$

The number of early children should be at a level where the marginal welfare cost from early fertility (the LHS) is exactly equal to the marginal welfare gain from early fertility (the RHS).

Regarding agents of type-2, three possible cases can arise, depending on the size of the cost differential between early and late children, and on the impact of early parenthood on future productivity. The three cases are: (1) $n_{a2} > 0$ and $n_{\ell 2} > 0$; (2) $n_{a2} = 0$ and $n_{\ell 2} > 0$; (3) $n_{a2} > 0$ and $n_{\ell 2} = 0$.

- If there exist some $n_{a2} > 0$ and $n_{\ell 2} > 0$ such that

  $$-u'(c_2)\, e_a + u'(d_2)\, w h_2'(n_{a2}) + v'(n_{a2} + n_{\ell 2}) = 0$$
  $$-u'(d_2)\, e_\ell + v'(n_{a2} + n_{\ell 2}) = 0$$

  then we have an interior solution for fertility in the two ages.[15] Simplifying those two conditions, we obtain:

  $$e_\ell - e_a = -w h_2'(n_{a2})$$

  Hence the marginal gain from early childbearing (the LHS), i.e. lower children costs, equals the marginal loss from early children (the RHS).

- Alternatively, when we have, for any $n_{a2} > 0$, the strict inequality:

  $$e_\ell - e_a < -w h_2'(n_{a2})$$

  then the marginal gain from early childbearing is necessarily inferior to the marginal loss from early children, so that the optimal number of early children is a corner solution, i.e. $n_{a2} = 0$. In that case, $n_{\ell 2}$ is given by:

  $$u'(d_2)\, e_\ell = v'(n_{\ell 2})$$

  That conditions equalizes the marginal loss from late childbearing (the LHS) with the marginal gain from late childbearing (the RHS). Hence, in that case, we have $n_{a2} = 0$ and $n_{\ell 2} > 0$.

[15] Obviously, given $e_a < e_\ell$, those two FOCs would not be all valid if $h_2'(n_{a2}) = 0$.

- Finally, when we have, for any $n_{a2} > 0$, the strict inequality:

$$e_\ell - e_a > -wh_2'(n_{a2})$$

then the marginal gain from early childbearing is necessarily superior to the marginal loss from early children, so that the optimal number of late children is a corner solution, i.e. $n_{\ell 2} = 0$. Then, n_{a2} is given by:

$$-u'(c_2)\, e_a + u'(d_2)\, wh_2'(n_{a2}) + v'(n_{a2}) = 0$$

Hence, in that case, we have $n_{a2} > 0$ and $n_{\ell 2} = 0$.

Regarding the *total* number of children, the number of early children for type-1 agents is given by:

$$u'(c_1)\, e_a = v'(n_{a1})$$

whereas, under $n_{a2} > 0$ and $n_{\ell 2} > 0$, we have

$$u'(c_2)\, e_a - u'(d_2)\, wh_2'(n_{a2}) = v'(n_{a2} + n_{\ell 2})$$

If $|h_2'(n_{a2})|$ is sufficiently large, type-2 agents have fewer children than type-1 agents: $n_{a2} + n_{\ell 2} < n_{a1}$.

When $n_{a2} = 0$, the optimal number of late children $n_{\ell 2}$ is given:

$$u'(d_2)\, e_\ell = v'(n_{\ell 2})$$

With a strongly concave $u(\cdot)$, it must be the case that $n_{\ell 2} < n_{a1}$.

Finally, when $n_{\ell 2} = 0$, the optimal number of early children n_{a2} satisfies:

$$u'(c_2)\, e_a - u'(d_2)\, wh_2'(n_{a2}) = v'(n_{a2})$$

Here again, if $|h_2'(n_{a2})|$ is large, we have, despite $c_2 > c_1$, that $n_{a2} < n_{a1}$.

In sum, whereas type-1 agents do not have late children, but only early children, three possible cases can arise for type-2 agents. If the cost differential between late and early parenthood is extremely large, as it used to be the case before the massive development of assisted reproductive technologies, then type-2 agents also opt for early children only. If, on the contrary, the cost differential between late and early is low, then type-2 agents have no early children, and only late children, since this will preserve their future productivity at little additional child costs. In the intermediate case, there is an interior solution with $n_{a2} > 0$ and $n_{\ell 2} > 0$, with a shape of fertility profile that depends on how sensitive the earnings profile is to early parenthood. Finally, the total number of children for type-1 agents tends to exceed the number of children for type-2 agents when the earnings profile of the latter is strongly sensitive to early births.

Therefore, although simple, the model developed in this section can explain the observed postponement of fertility as a result of the development of assisted reproductive technologies, which reduced the gap $e_\ell - e_a$. Our model allows us also to rationalize the observed heterogeneity: in conformity with empirical studies (Cigno and Ermisch 1989, Ermisch and Ogawa 1995, Joshi 2002), adults with lower career opportunities tend to make children earlier than adults with (potentially) steeper earnings profiles. Finally, our framework can also, under general conditions, explain why the tendency towards steeper potential earnings profiles over time tends to reduce total fertility.

3 The first-best problem

Let us now characterize the social optimum of our economy. For that purpose, we will focus on a classical utilitarian social objective, whose goal coincides with the maximization of aggregate welfare. We also focus here on the aggregate welfare of a single cohort of individuals, which includes a fixed fraction π_1 of agents of type 1, and a fixed fraction π_2 of agents of type 2. Whereas focusing on a single cohort of given size is an obvious simplification, this allows us to escape from well-known difficulties raised by population ethics.[16]

The utilitarian first-best optimum is obtained by maximizing the following Lagrangian:

$$\sum \pi_i \left(U_i - \mu \left[c_i + d_i + b_i + n_{ai} e_a + n_{\ell i} e_\ell - w \left(1 + h_i \left(e_{ai}\right)\right)\right]\right)$$

where μ denotes the Lagrange multiplier associated with the resource constraint of the economy.

Assuming interior solutions, we obtain the following conditions:

$$\begin{aligned} u'\left(c_i\right) = u'\left(d_i\right) = u'\left(b_i\right) &= \mu \\ v'\left(n_{ai} + n_{\ell i}\right) &= \mu \left[e_a - w h_i'\left(n_{ai}\right)\right] \\ v'\left(n_{ai} + n_{\ell i}\right) &= \mu e_\ell \end{aligned}$$

The utilitarian social optimum involves an equalization of all consumptions across individuals of all types, and across periods. Thus, whatever agents have an increasing productivity profile (i.e. type-2 agents) or a flat productivity profile (i.e. type-1 agents), they should all enjoy the same consumptions at all periods of their life.

Regarding optimal fertility, we have, for type-1 agents, that the FOCs for n_{a1} and $n_{\ell 1}$ cannot be both satisfied. Given $e_\ell > e_a$, the social optimum involves, as in the laissez-faire, $n_{a1} > 0$ and $n_{\ell 1} = 0$, that is, it is not socially optimal that type-1 agents have late children. As far as type-2 agents are concerned, three cases can arise, like at the laissez-faire.

If there exists some $n_{a2} > 0$ such that:

$$e_\ell - e_a = -w h_2'\left(n_{a2}\right)$$

then the FOCs for n_{a2} and $n_{\ell 2}$ can be simultaneously valid. As a consequence, we have: $n_{a2} > 0$ and $n_{\ell 2} > 0$ at the social optimum.

[16] On this, see Blackorby *et al.* (2005).

On the contrary, if, for any $n_{a2} > 0$, we have

$$e_\ell - e_a < -wh_2'(n_{a2})$$

then we have $n_{a2} = 0$ and $n_{\ell 2} > 0$ at the social optimum.

Finally, if, for any $n_{a2} > 0$, we have

$$e_\ell - e_a > -wh_2'(n_{a2})$$

then we have $n_{a2} > 0$ and $n_{\ell 2} = 0$ at the social optimum.

Regarding the *total* number of children, we have, for agents of types 1:

$$v'(n_{a1}) = \mu e_a$$

whereas, type-2 agents's fertility satisfies one of the following conditions:

$$\begin{aligned} v'(n_{a2} + n_{\ell 2}) &= \mu e_\ell && \text{if } n_{\ell 2} > 0 \\ v'(n_{a2}) &= \mu\left[e_a - wh_2'(n_{a2})\right] && \text{if } n_{\ell 2} = 0 \end{aligned}$$

Thus, provided the optimal number of late children for type-2 agents is positive, we obtain, given $e_\ell > e_a$, that the optimal number of children for type-2 agents is lower than the optimal number of children for type-1 agents:

$$n_{a1} > n_{a2} + n_{\ell 2}$$

The same result holds if the optimal number of late children for type-2 agents is zero. Indeed, in that case, we obtain, given $wh_2'(n_{a2}) < 0$, that the optimal number of children for type-2 agents is lower than the optimal number of children for type-1 agents:

$$n_{a1} > n_{a2}$$

Thus utilitarianism recommends the equalization of consumptions across individuals and time periods, as well as a larger total fertility for type-1 agents than for type-2 agents. The social optimum involves only early children for type-1 agents, and possibly early and/or late children for type-2 agents, depending on the cost gap between late and early children, and on the sensitivity of productivity to early parenthood.

Hence, the utilitarian social optimum differs significantly from the laissez-faire. Whereas type-1 agents have lower consumptions, and may even have fewer children than type-2 agents at the laissez-faire, the utilitarian optimum recommends the same consumptions for all agents, as well as more children for type-1 agents than for type-2 agents.

Regarding the decentralization of the social optimum, this can be achieved by means of lump-sum transfers from type-2 agents towards type-1 agents, in such a way as to equalize all consumptions. Note that such transfers, by reducing the marginal utility of consumption for type-1 agents, will raise early fertility n_{a1} above its laissez-faire level. Inversely, by increasing the marginal utility of consumption for type-2 agents, those transfers will tend to reduce the fertility of type-2 agents, but without affecting the timing of births.[17]

[17] The conditions describing the type of optimum (interior or corner) are independent from consumption levels, and depend only on e_a, e_ℓ and $h_2'(\cdot)$.

4 Family allowances

The previous section showed how the first-best optimum can be decentralized by means of lump sum transfers from individuals with high earnings potential towards individuals with low earnings potential. However, such an intervention requires both that lump sum transfers are available policy instruments, and that the government can observe the types of individuals. These are strong assumptions. In this section, we depart from such a first-best problem, and consider instead the characterization of the second-best social optimum.

For that purpose, we will proceed in two stages. We will first focus on the optimal policy when the only available fiscal instruments are *uniform* payroll taxes, demogrants and children allowances. Then, we will consider the optimal policy under asymmetric information, under non-linear fiscal instruments.

4.1 Linear case

The government has, as available policy instruments, a uniform payroll tax, τ, a uniform demogrant, T, and a uniform subsidy on the number of children, σ. The instrument σ can be understood as a family allowance.

We assume, as above, that the two types of agents differ regarding the slope of their (potential) earning profile: $h_2(n_{a2}) > h_1(n_{a1}) \geq 1$, with $h_i'(n_{ai}) < 0$. However, given that we are here mainly concerned with the timing of fertility rather than with the total fertility, we assume that, for each individual, the total fertility is fixed to its replacement level, i.e. $n_{ai} + n_{\ell i} = 1$. To simplify notations, we denote early fertility n_{ai} by $n_i = 1 - n_{\ell i}$, and we normalize the utility from children, in such a way that $v(1) = 0$.

In the laissez-faire, each agent of type $i \in \{1, 2\}$ maximizes:

$$\begin{aligned} U_i &= u_i(c_i) + u(d_i) + u(b_i) - \mu\left[c_i + d_i + b_i - T - (1-\tau)\right. \\ &\qquad \left. w(1 + h_i(n_i)) + (1-\sigma)((e_a - e_\ell)n_i + e_\ell)\right]. \end{aligned}$$

where μ is the Lagrange multiplier associated with the agent's budget constraint.

The FOCs are:

$$u_i'(c_i) = u'(d_i) = u'(b_i) = \mu$$
$$-(1-\tau)wh_i'(n_i) + (1-\sigma)(e_a - e_\ell) = 0$$

Thus agents tend, here again, to smooth consumption across periods, and organize the timing of births in such a way as to equalize the marginal welfare losses and marginal welfare gains from early parenthood.

As to the government, its objective is to maximize the sum of individual utilities subject to its revenue constraint. Namely, it maximizes the following Lagrangian:

$$\begin{aligned} \mathcal{L} &= \sum \pi_i \left\{ 3u\left(\frac{(1-\tau)w(1+h_i(n_i)) + T - (1-\sigma)((e_a - e_\ell)n_i + e_\ell)}{3}\right)\right. \\ &\quad \left. +\mu\left[\tau w(1 + h_i(n_i)) - T - \sigma(e_a - e_\ell)n_i + \sigma e_\ell\right]\right\} \end{aligned}$$

where μ is the Lagrange multiplier associated with the revenue constraint.

Using the envelope theorem, we obtain the following FOCs:[18]

$$\frac{\partial \mathcal{L}}{\partial \tau} = \begin{array}{c} -\sum \pi_i \left[u'(x_i) w (1+h_i(n_i)) - \mu w (1+h_i(n_i)) \right. \\ \left. -\mu\tau w h_i'(n_i) \frac{\partial n_i}{\partial \tau} - \mu\sigma (e_a - e_\ell) \frac{\partial n_i}{\partial \tau} \right] \end{array} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \sigma} = \begin{array}{c} \sum \pi_i \left[u'(x_i) \left[(e_a - e_\ell) n_i + e_\ell \right] + \mu\tau w h_i'(n_i) \frac{\partial n_i}{\partial \sigma} \right. \\ \left. -\mu\sigma (e_a - e_\ell) \frac{\partial n_i}{\partial \sigma} - \mu \left[(e_a - e_\ell) n_i + e_\ell \right] \right] \end{array} = 0$$

$$\frac{\partial \mathcal{L}}{\partial T} = \begin{array}{c} \sum \pi_i \left[u'(x_i) - \mu (1+\sigma)(e_a - e_\ell) \frac{\partial n_i}{\partial T} \right. \\ \left. +\tau w h_i'(n_i) \frac{\partial n_i}{\partial T} \right] \end{array} = 0$$

where $x_i \equiv \frac{(1-\tau)w(1+h_i(n_i))+T-(1-\sigma)((e_a-e_\ell)n_i+e_\ell)}{3}$ is the argument of the agent i's temporal utility $u(\cdot)$.

Those conditions characterize the optimal values for our instruments τ, σ and T. Note, however, that the simultaneous study of the optimal levels of the three taxation tools would be quite laborious, as their values are related to each others through the government's budget constraint. Hence, to keep the analysis simple, we will proceed as follows. To interpret those optimality conditions, we will consider alternative pairs of instruments, holding the other instrument equal to 0. Hence, we will focus on the pairs (τ, T) and (σ, T), while keeping, each time, the other fiscal tool set to 0. This will allow us to derive, *in fine*, closed-form solutions for the optimal levels of instruments.

Let us start with the pair (τ, T), composed of a payroll tax on labor earnings and a first-period demogrant. The first FOC from above does not suffice, on its own, to characterize the optimal level of τ, as a rise in τ must, under the government's budget constraint, imply a change in the demogrant T, in such a way as to maintain the budget equilibrium. Therefore, in order to characterize the optimal τ, we will use a compensated Lagrangian expression, whose derivative with respect to the policy instrument τ gives us the effect of a variation of τ on the Lagrangian when that change is compensated by a variation of T that keeps the government's budget equilibrium. Using the optimality conditions, the derivative of the compensated Lagrangian can be defined as:

$$\begin{aligned} \frac{\partial \tilde{\mathcal{L}}}{\partial \tau} &\equiv \frac{\partial \mathcal{L}}{\partial \tau} + \frac{\partial \mathcal{L}}{\partial T}\frac{\partial T}{\partial \tau} \\ &= \frac{\partial \mathcal{L}}{\partial \tau} + \frac{\partial \mathcal{L}}{\partial T} E\left[w (1+h(n)) \right] \end{aligned}$$

where $\tilde{\mathcal{L}}$ denotes the compensated Lagrangian, and where the second term of the RHS accounts for the effect of a change in the tax rate τ on the first-period demogrant T, under the government's budget equilibrium constraint. The operator $E(\cdot)$ denotes the *average* value of its argument among the population.

[18] For simplicity we assume interior solutions, which excludes the case where $h_1 = 1$.

Substituting for the FOCs for optimal τ and T and equalizing to zero yields:

$$\begin{aligned}\frac{\partial\tilde{\mathcal{L}}}{\partial\tau} &= \frac{\partial\mathcal{L}}{\partial\tau}+\frac{\partial\mathcal{L}}{\partial T}E\left[w\left(1+h\left(n\right)\right)\right] \\ &= -E\left[u'\left(x\right)w\left(1+h\left(n\right)\right)\right]+E\left[u'\left(x\right)w\right]E\left[\left(1+h\left(n\right)\right)\right]-\mu E\left(A\frac{\partial\tilde{n}}{\partial\tau}\right)=0\end{aligned}$$

where $A\equiv\tau wh'\left(n\right)+\sigma\left(e_{\ell}-e_{a}\right)$ is the effect of n on the revenue constraint (the first term is negative and the second positive) and $\frac{\partial\tilde{n}}{\partial\tau}\equiv\frac{\partial n}{\partial\tau}+\frac{\partial n}{\partial T}\frac{\partial T}{\partial\tau}$ denotes the effect of a change in τ on early fertility, when that change is compensated by a change in T so as to maintain the budget equilibrium.

Regarding the pair (σ,T), one can proceed in the same way as with the pair (τ,T), and define the derivative of the compensated Lagrangian as follows:

$$\begin{aligned}\frac{\partial\tilde{\mathcal{L}}}{\partial\sigma} &\equiv \frac{\partial\mathcal{L}}{\partial\sigma}+\frac{\partial\mathcal{L}}{\partial T}\frac{\partial T}{\partial\sigma} \\ &= \frac{\partial\mathcal{L}}{\partial\sigma}-\frac{\partial\mathcal{L}}{\partial T}\left[\left(e_{a}-e_{\ell}\right)E\left(n\right)-e_{\ell}\right]\end{aligned}$$

where the second term is the effect of a change in σ on the demogrant, under the government's budget equilibrium.

Substituting for the FOCs for optimal σ and T and equalizing to zero yields:

$$\begin{aligned}\frac{\partial\tilde{\mathcal{L}}}{\partial\sigma} &= \frac{\partial\mathcal{L}}{\partial\sigma}-\frac{\partial\mathcal{L}}{\partial T}\left[\left(e_{a}-e_{\ell}\right)E\left(n\right)-e_{\ell}\right] \\ &= \left(e_{a}-e_{\ell}\right)\left[E\left(u'\left(x\right)n\right)-E\left(u'\left(x\right)\right)E\left(n\right)\right]+\mu E\left(A\frac{\partial\tilde{n}}{\partial\sigma}\right)=0\end{aligned}$$

where $\frac{\partial\tilde{n}}{\partial\sigma}\equiv\frac{\partial n}{\partial\sigma}+\frac{\partial n}{\partial T}\frac{\partial T}{\partial\sigma}$ denotes the effect of a change in σ on early fertility when that change is compensated by a change in T in such a way as to maintain the budget equilibrium.

The two compensated Lagrangian conditions can be rewritten as:

$$\begin{aligned}-cov\left(u'\left(x\right),\left(1+h\left(n\right)\right)\right)-\mu E\left(A\frac{\partial\tilde{n}}{\partial\tau}\right) &= 0 \\ cov\left(u'\left(x\right),n\right)\left(e_{a}-e_{\ell}\right)+\mu E\left(A\frac{\partial\tilde{n}}{\partial\sigma}\right) &= 0\end{aligned}$$

To interpret these tax formulae, we look at pairs of instruments : τ and T and σ and T. Then, we obtain the following expressions for the optimal payroll tax and the optimal child subsidy:

$$\tau=\frac{-cov\left(u'\left(x\right),w\left(1+h\left(n\right)\right)\right)}{\mu E\left(\frac{\partial\tilde{n}}{\partial\tau}h'\left(n\right)\right)} \tag{5}$$

$$\sigma=\frac{cov\left(u'\left(x\right),n\right)}{\mu E\left(\frac{\partial\tilde{n}}{\partial\sigma}\right)} \tag{6}$$

Regarding the optimal payroll tax, note first that the numerator of (5) is positive, as the covariance is negative, the correlation between x and $h(n)$ being positive, given $h'(n) < 0$. This is the standard equity term of the tax formula. However, the sign of τ depends also on that of the denominator, whose sign depends on the one of $-\partial\tilde{n}/\partial\tau$. If the (aggregate) compensated effect of the tax on n is negative, then the tax is desirable. This is the efficiency term of the tax formula. On the contrary, if the (aggregate) compensated effect of τ on n is positive, the denominator is negative, implying a negative optimal τ.

As far as optimal family allowances are concerned, the numerator of (6) is positive (low income agents have early children). The equity term of the fiscal formula pushes towards the subsidization of children. We thus have positive child benefits if the denominator is also positive, that is, if those children benefits have a positive (aggregate) effect on early childbearing in compensated terms.

In sum, the optimal policy consists, in the absence of lump sum transfers, into a subsidization of children, to the extent that such family allowances increase early parenthood. However, if subsidizing children reduces early parenthood, then family allowances are no longer justified, and a taxation of children is required instead. The intuition is the following. If subsidizing children fosters early parenthood, this means, given that early parenthood concerns generally the individuals with low career opportunities, that family allowances are a way to redistribute resources in the right direction (i.e. individuals with low career opportunities), by subsidizing a good that those agents consume more than others. Inversely, if subsidizing children reduces early parenthood and raises late parenthood, family allowances are regressive, since these amount to subsidize a good that is mostly consumed by individuals with high career opportunities.

Finally, note that, as an alternative to this presentation, we could replace the *general* children subsidy σ by a subsidy σ_a on *early* childbearing only.[19] Under such a refined instrument, children would be subsidized differently, according to the age of their parents. In that case, the optimal child allowance would be:

$$\sigma_a = \frac{cov\left(u'\left(x\right), n\right)}{\mu E \dfrac{\partial\tilde{n}}{\partial\sigma_a}}$$

where $\partial\tilde{n}/\partial\sigma_a$ is the effect, on early fertility, of a change in σ_a compensated by a change in T in such a way as to maintain the budget equilibrium. The compensated effect $\partial\tilde{n}/\partial\sigma_a$ is positive, leading to a subsidy on early children.

Hence, the availability of children allowances differentiated according to the age of parents would be an indirect way to redistribute resources towards agents with low career opportunities, who are usually the ones who have early children.

4.2 Non linear case: 2 types

Let us now turn to an alternative formulation of the second-best problem, where the available fiscal instruments are not restricted to linear (uniform) instruments, but where the government cannot observe the types of agents. Here

[19] In that case, there is no tax nor subsidy on late childbearing.

again, we assume, as above, a perfect capital market with zero rate of interest and no time preference.

The problem for an individual of type i is to maximize:

$$u(c_i) + u(d_i) + u(b_i) - \mu\left[c_i + d_i + b_i - w(1 + h_i(n_i)) + n_i(e_a - e_\ell) + e_\ell\right].$$

This implies $c_i = d_i = b_i$, so that the utility can be reduced to

$$\varphi\left[w(1 + h_i(n_i)) - n_i(e_a - e_\ell) - e_\ell + T_i\right]$$

where $\varphi(\cdot)$ denotes the agent's lifetime welfare ($\varphi'(\cdot) > 0$, $\varphi''(\cdot) < 0$), whereas T_i denotes a lump sum transfer such that $\sum \pi_i T_i = 0$.

The central planner cannot observe the types of agents. We assume that $h_2(n) > h_1(n) \geqslant 1$ with $h_i'(n) < 0$. If the difference between h_2 and h_1 is big enough, type-1 agents will choose a very high value of n_1, and type-2 agents will postpone childbearing. In the first-best, there will be some redistribution from type 2 to type 1, namely $T_1 > 0 > T_2$.

In the second-best, the central planner cannot observe the types of agents. Hence, it is tempting, for an agent of type 2, that is, with high career opportunities, to pretend to be of type 1, in such a way as to benefit from public transfers. Therefore, the social planner must make sure that type-2 agents do not mimic type-1 agents. This leads us to the self-selection constraint:

$$\begin{aligned}&\varphi\left[w(1 + h_2(n_2)) + n_2(e_\ell - e_a) - e_\ell + T_2\right] \geqslant \\ &\varphi\left[w(1 + h_2(n_1)) + n_1(e_\ell - e_a) - e_\ell + T_1\right].\end{aligned} \tag{7}$$

It is clear that, if we had assumed $h_1(n_1) = 1$ implying $n_1 = 1$, then

$$T_1 = \pi_2\left(wh_2(n_2) + n_2(e_a - e_\ell)\right)$$

which is the first-best solution.

Formally, the second-best problem can be written by means of the Lagrangian:

$$\begin{aligned}\mathcal{L} &= \sum \pi_i\left\{\varphi\left(w(1 + h_i(n_i)) + n_i(e_\ell - e_a) - e_\ell + T_i\right) - \gamma T_i\right\} \\ &\quad + \lambda\left\{\begin{array}{l}\varphi\left(w(1 + h_2(n_2)) + n_2(e_\ell - e_a) - e_\ell + T_2\right) \\ -\varphi\left(w(1 + h_2(n_1)) + n_1(e_\ell - e_a) - e_\ell + T_1\right)\end{array}\right\}\end{aligned}$$

where γ is the Lagrange multiplier associated with the constraint on lump sum transfers, whereas λ is the Lagrange multiplier associated with the self-selection constraint.

The FOCs are:

- $n_1 : \pi_1\varphi'(x_1)\left[wh_1'(n_1) + (e_\ell - e_a)\right] - \lambda\varphi'(x_2)\left[wh_2'(n_1) + (e_\ell - e_a)\right] = 0$
- $n_2 : \left[\varphi'(x_2)\pi_2 + \lambda\right]\left[wh_2'(n_2) + (e_\ell - e_a)\right] = 0$
- $T_1 : \pi_1(\varphi'(x_1) - \gamma) - \lambda\varphi'(x_2) = 0$

- $T_2 : \pi_2 (\varphi' (x_2) - \gamma) + \lambda \varphi' (x_2) = 0$

First of all, we see that, when the self-selection constraint is not binding ($\lambda = 0$), we have the first-best outcome:

$$e_\ell - e_a = -wh'_i (n_i)$$
$$\varphi' (x_i) = \gamma$$

On the contrary, when $\lambda > 0$, we have:

$$\begin{aligned} e_\ell - e_a &= -wh'_2 (n_2) \\ \varphi' (x_1) &> \gamma > \varphi' (x_2) \end{aligned}$$

In other words, in the choice of n_2, one has the standard "non distortion at the top" result. As to redistribution, it is only partial, since we have: $x_2 > x_1$. Thus, the presence of asymmetric information prevents the equalization of lifetime welfare across types, unlike in the first-best optimum. Finally, we have

$$wh'_1 + (e_\ell - e_a) = \lambda \frac{\varphi' (\tilde{x}_2)}{\varphi' (x_1) \pi_1} [wh'_2 (n_1) + (e_\ell - e_a)] .$$

This expression means that one should have a subsidy on n_1 to relax the self-selection constraint. Indeed, given that early children are more important for agents of type 1 in comparison to agents of type 2 (for whom early childbearing has worse effects on the earnings profile), a simple way to prevent type-2 agents from pretending to be of type 1 to get transfers consists of subsidizing children.

In sum, the optimal policy under asymmetric information involves children allowances that are differentiated according to the parents' (potential) earnings profiles. Whereas early children from parents with (potentially) steep earnings profiles are not subsidized, early children from parents with flat earnings profile should be subsidized, as a way to solve the self-selection problem. Thus, even if the government cannot observe the type of agents, it can nonetheless insure the self-selection of types by proposing different children allowances, which make, by construction, any mimicking suboptimal for individuals.

5 Childbearing and PAYG social security

Up to now, we studied the optimal family policy under a perfect capital market. We will now relax that assumption. For that purpose, we will assume that the only way individuals can provide resources for their old age is through a contract such that when retired they get a fraction of their children's earnings.

Such a contract can take various forms. It can be a standard Pay-As-You-Go pension system (PAYG), which provides a pension to the elderly thanks to the contributions of the young active individuals. It can also take the form of intergenerational trade within the (extended) family, each active child giving some fraction of his income to the inactive elderly in his family. Moreover, the system can be an *individualized* system, where parents internalize the effect of their fertility choices on retirement benefits, or, alternatively, a *collective* system, where parents can free-ride on the system (either PAYG or familial), to get large pensions without supporting the cost of fertility.

As we shall see, relaxing the perfect capital market assumption and replacing it by a pension system is not neutral at all for the design of the optimal family allowances in the context of endogenous childbearing ages. The reason is that, once a (formal or informal) pension system is introduced, there might be, for parents, an incentive in having *early* children. The reasons are twofold. First, children, if born a longer time ago, can, under increasing earnings profiles, provide higher pensions to their parents. Second, and more importantly, when parents with early children are retired, they can count not on one, but on *two* generations of workers: their own children *and* their grandchildren.

To illustrate this, we will first, for the sake of presentation, study fertility choices under an individualized pension system, when there is only one type of agents. We will then reintroduce heterogeneity later on in this section.

5.1 Laissez-faire and optimal policy with one type

In that context, the problem faced by an individual amounts to choosing savings s_a, early children n_a, late children n_ℓ and contribution rate γ to maximize:[20]

$$\begin{aligned} U &= u\left[(1-\gamma)\,w - e_a n_a - s_a\right] + u\left[(1-\gamma)\,wh\left(n_a\right) - e_\ell n_\ell + s_a\right] \\ &\quad + u\left[\gamma\left(wn_\ell + wh\left(\bar{n}_a\right)n_a + wn_a\bar{n}_a\right)\right] + v\left(n_a+n_\ell\right) \end{aligned}$$

where γ is the fraction of earnings that is paid to the elderly, while $\bar{n}_a$ means that the individual expects his early children to have made the same number of early children as he made himself (i.e. n_a), without having any control on it. Note that the earnings' basis available for the funding of the elderly's pensions includes three terms: the first two terms (i.e. wn_ℓ and $wh\left(\bar{n}_a\right)n_a$) concern the incomes from the late and the early children, whereas the third term, i.e. $wn_a\bar{n}_a$, concerns the income of grandchildren, whose number is $n_a\bar{n}_a$ (as each of the n_a early child had also $\bar{n}_a$ early children once adult).

We can thus obtain the following FOCs:

$$s_a \;:\; u'\left(c\right) = u'\left(d\right) \tag{8}$$

$$\gamma \;:\; u'\left(c\right) = u'\left(b\right)\frac{n_\ell + h\left(\bar{n}_a\right)n_a + n_a\bar{n}_a}{1+h\left(n_a\right)} \tag{9}$$

$$n_\ell \;:\; u'\left(c\right)e_\ell = u'\left(b\right)\gamma w + v'\left(n_a+n_\ell\right) \tag{10}$$

$$n_a \;:\; u'\left(c\right)\left[e_a + (1-\gamma)\,wh'\left(n_a\right)\right] = u'\left(b\right)\gamma\left(wh\left(\bar{n}_a\right) + w\bar{n}_a\right) + v'\left(n_a+n_\ell\right) \tag{11}$$

In a world of identical individuals, these 4 conditions lead to the optimal decision concerning s_a, n_a, n_ℓ and γ as long as each individual considers that the

[20] To keep things simple, we assume that savings between the first and the second periods is possible.

pension scheme operates at the level of the family. It is interesting to observe that such a PAYG scheme creates a distortion in favor of early childbearing, relative to a setting of a fully funded pension system. The reason is that early children have higher earnings when their parents retire, and that they have themselves some working children, unlike the parents' late children.

If, instead of that individualized pension system, the PAYG scheme were collective, each agent would not, when making his fertility choices, perceive his impact on pension benefits. Hence, agents would choose a *lower* level of fertility. To see this, note that, under collective pensions, conditions (8) and (9) would be unchanged, but conditions (10) and (11) would become:

$$u'(c)\, e_\ell = v'(n_a + n_\ell) \tag{10'}$$
$$u'(c)\left[e_a - (1-\gamma)\, wh'(n_a)\right] = v'(n_a + n_\ell) \tag{11'}$$

Given that the RHS of (10) includes an additional positive term in comparison to (10'), agents tend to make, under a collective pensions system, *fewer* late children n_ℓ than under an individualized system. Similarly, the RHS of (11) includes an additional positive term absent in (11'), so that the collective pensions system induces *fewer* early children than under individualized pensions.

Therefore, in order to induce the optimal fertility behavior under a collective system, one would need Pigouvian subsidies σ_a and σ_ℓ on early and late children, such that

$$\begin{aligned} \sigma_a &= w\gamma\left(h(\bar{n}_a) + \bar{n}_a\right)\frac{u'(b)}{u'(c)} \\ \sigma_\ell &= w\gamma\frac{u'(b)}{u'(c)} \end{aligned}$$

This is a standard problem with PAYG and fertility (see Gahavari 2009): individuals do not internalize the effect that their fertility choice can have on pension benefits. We have the following inequality between Pigouvian subsidies:

$$\sigma_a > \sigma_\ell$$

The children subsidy should be higher on early than on late childbearing, even though one can have $n_\ell > n_a$. There are two reasons why the external effects of early fertility exceed the ones of late fertility. First, under $h(\bar{n}_a) > 1$, the *levels* of earnings of contributors born from young parents are larger than the earnings of contributors born from older parents, since only the former can benefit from increasing productivity. Second, the *number* of contributors is strongly affected by early fertility decisions, through the number of grandchildren. On the contrary, in the case of late parenthood, grandchildren cannot fund the pension system, and so the fertility externality is less sizeable.

Those arguments tend to justify a differentiated fiscal treatment of early and late parenthoods, under the form of children allowances that depend on the age of parents. Thus, even if one assumes a perfect homogeneity of the population in terms of career opportunity and earnings profile, there is already a strong argument not only for subsidizing children, but also for subsidizing them differently depending on the age of their parents. The justification is that fertility externalities induced by early and late fertility are not of the same magnitudes, inviting distinct Pigouvian subsidies.

5.2 Laissez-faire and optimal policy under two types

Whereas the above policy analysis was carried out for a *unique* type of agents, let us now turn back to the case when individuals differ in their future career and earnings opportunities (i.e. the future earnings function $h_i(n_{ai})$). For that purpose, we assume, here again, $h_2(n_{a2}) > h_1(n_{a1}) \geq 1$, with $h_i'(n_{ai}) < 0$. For simplicity of presentation, we suppose also that $n_{ai} = n_i = 1 - n_{\ell i}$, and that $v(1) = 0$. We also assume saving between first and second period.

Given that children allowances differentiated according to the age of parents have been already studied above, we will abstract here from these, and assume that the available fiscal instruments are: a uniform child benefit of rate σ, a payroll tax τ and flat rate PAYG pension p. Given those assumptions, we have:

$$c_i = d_i$$

and thus

$$U_i = 2u\left(\frac{(1-\tau)\,w\,(1+h_i\,(n_i)) - (1-\sigma)\,(n_i\,(e_a - e_\ell) - e_\ell)}{2}\right) + u\,(p)\,.$$

We also have:

$$p = \sum \pi_i \left[\tau w \left(1 - n_i + h_i\,(n_i)\,n_i + n_i^2\right) - \sigma\,(e_a n_i + e_\ell\,(1-n_i))\right].$$

Each individual $i \in \{1,2\}$ chooses n_i without seeing the effect his choice has on his future pension benefits. His choice of n_i is determined by the FOC:

$$u'\,(c_i)\,[w\,(1-\tau)\,h_i'\,(n_i) - (1-\sigma)\,(e_a - e_\ell)] = 0.$$

That condition does not yield the socially optimal level of early children, since this neglects the impact of early fertility on old-age pensions through its impact on the budget constraint of the economy.

To see this, let us now turn to the social planner's problem. We can rewrite it by means of the following Lagrangian:

$$\mathcal{L} = \sum \pi_i \left\{U_i - \mu\left[p + \sigma\,(e_a n_i + e_\ell\,(1-n_i)) - \tau w \left(1 - n_i + h_i\,(n_i)\,n_i + n_i^2\right)\right]\right\}.$$

where μ is the Lagrange multiplier associated with the government's budget constraint.

Using again the operator E to lighten the presentation, we have the FOCs:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial p} &= E\,[u'\,(p) - \mu] = 0 \\
\frac{\partial \mathcal{L}}{\partial \tau} &= -E\,(u'\,(c)\,w\,(1+h\,(n))) + \mu E\left(w\left(1 - n + h\,(n)\,n + n^2\right)\right) - \mu E\left(B\frac{\partial n}{\partial \tau}\right) = 0 \\
\frac{\partial \mathcal{L}}{\partial \sigma} &= E\,[u'\,(c)\,[n\,(e_a - e_\ell) - e_\ell]] - \mu E\,((e_a n + e_\ell\,(1+n))) - \mu E\left(B\frac{\partial n}{\partial \sigma}\right) = 0
\end{aligned}$$

where $B \equiv \sigma(e_a - e_\ell) - \tau w(-1 + h(n) + nh'(n) + 2n)$ denotes the effect of n on the budget constraint. A marginal increase in early fertility n has 4 revenue effects: (1) a revenue increase thanks to lower child cost (i.e. as $e_a < e_\ell$); (2) a revenue loss as $h'(n) < 0$ (due to the productivity loss induced by early childbearing); (3) another revenue gain as $h > 1$ (thanks to the larger contributions of the children once older and more productive); (4) and another revenue gain through grandchildren (which is linear in the number of early children). A sufficient condition for $B < 0$ is: $-\frac{nh'(n)}{h} < 1 + \frac{2n-1}{h}$.

Those FOCs can be rewritten as:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau} &= -cov(u'(x), 1+h) + E[u'(p) - v'(x)]E(1+h) \\ &\quad + u'(p)E\left(1 - n + h(n)n + n^2\right) - \frac{E(u'(p))}{w}E\left(B\frac{\partial n}{\partial \tau}\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \sigma} &= (e_a - e_\ell)\, cov[u'(x)] - E(u'(b) - u'(x))E(n) \\ &\quad - E(u'(b))E\left(B\frac{\partial n}{\partial \sigma}\right).\end{aligned}$$

To better interpret these expressions, let us consider the case where only pairs of fiscal instruments are available: on the one hand, the pair (τ, p), that is, only a tax rate and a pension are available instruments; on the other hand, the pair (σ, p), that is, only a child allowance and a pension are available instruments.

Let us first consider the pair (τ, p), σ being fixed to 0. Equalizing the above FOCs to 0 and isolating τ yields:

$$\tau = \frac{-cov\left(u'(x), 1+h\right) + E\left(u'(p) - u'(c)\right)E(1+h) + u'(p)E\left(1 - n + h(n) + n^2\right)}{\frac{E\left(u'(p)\right)}{w}E\left(\frac{\partial n}{\partial \tau}(-1 + nh' + h + 2n)\right)}$$

That formula defines the optimal level of the payroll tax τ when the fiscal revenues are used to fund a pension system only. The numerator consists of 3 terms that are all expected to be positive (assuming that $E(u'(p) - u'(c)) > 0$). Hence those three terms push toward more redistribution through the income tax. The denominator gives the effect of the tax on public revenue. The sign of this efficiency effect depends on the impact of earnings taxation on aggregate early fertility (i.e. $E(\partial n/\partial \tau)$), and on the impact of aggregate early fertility on the government's budget constraint. If the impact of τ on the aggregate early fertility is negative (i.e. $E(\partial n/\partial \tau) < 0$), and if the impact of early fertility on fiscal revenues is also negative, (i.e. if $\frac{nh'(n)}{h} < -1 - \frac{2n-1}{h}$), the efficiency effect is also positive. As a consequence, the optimal tax is positive: $\tau > 0$.

Turning now to the pair (σ, p), under the assumption $\tau = 0$. Equalizing the above FOCs to 0 and isolating σ yields:

$$\sigma = \frac{(e_\ell - e_a)\, cov(u'(x), n) - E(u'(p) - u'(c))}{(e_\ell - e_a)\, Eu'(p)\, E\frac{\partial n}{\partial \sigma}}.$$

Assume that the poor (low h_i) tend to have more early children than the rich (i.e. $dn/dh < 0$). Assume further that child allowances have a positive effect on (total) early fertility (i.e. $E(\partial n/\partial \sigma) > 0$). Then the sign of the optimal child allowance rate σ depends on the relative strength of the two terms of the numerator. On the one hand, the positive covariance term pushes for a positive σ and a low pension. Indeed, if the poor tend to make more children, the equity term pushes towards the subsidization of children, i.e. $\sigma > 0$. That effect is increasing in the cost differential between late and early childbearing, i.e. $(e_\ell - e_a)$. On the other hand, the positive term $Eu'(p) - Eu'(c)$ pushes towards a negative σ aimed at financing the pension scheme. Hence, the sign of the optimal child allowance depends on the strengths of those two terms.

In sum, once we reintroduce heterogeneity in terms of earnings profile, the optimal family policy requires an arbitrage between equity and efficiency. Given that agents with lower future career opportunities tend to make children earlier, the subsidization of fertility is supported by equity concerns. The cost gap between late and early fertility tends to reinforce that equity case for children allowances. However, a higher cost gap between late and early childbearing pushes, from an efficiency point of view, towards lower family allowances. As a consequence, the observed falling cost gap between late and early childbearing has a somewhat ambiguous effect on the optimal family allowances.

6 Conclusion

As this is widely acknowledged among demographers, European economies have been characterized, during the last three decades, by a tendency towards the postponement of fertility. Through its consequences on the economy as a whole, that demographic trend raises various challenges to policy-makers.

The present paper aimed at examining the design of the optimal family policy in an economy where not only total fertility, but also the timing of births, are chosen by individuals. For that purpose, we developed a simple three-period model, where the adult population can have children either in young adulthood, or in older adulthood, and where late childbearing, although possible, is more costly than early childbearing. Moreover, the adult population was assumed to be partitioned in two groups, some individuals having promising earnings opportunities in the future (conditionally on the number of children at the young age), whereas other individuals face a flat earnings profile.

That simple model explains the observed postponement of fertility as a result of the development of assisted reproductive technologies, which reduced the cost gap between late and early childbearing. Our model allows us also to rationalize the observed heterogeneity in terms of childbearing age, that is, the stylized fact according to which adults with lower career opportunities tend to make children earlier than adults with steeper earnings profiles.

Regarding the optimal public intervention, we firstly considered the first-best utilitarian optimum, and showed that this can be decentralized by means of lump-sum transfers from increasing-productivity agents towards constant-productivity agents, in such a way as to equalize consumptions at all periods for all agents. Such transfers were shown to raise constant-productivity individuals' early fertility above its laissez-faire level, and to reduce the increasing-productivity individuals' late fertility, but without affecting the timing of births for the two types.

Then, considering an economy where only linear uniform policy instruments are available, we showed that the optimal second-best policy involves a subsidization of children, to the extent that it fosters early parenthood. Indeed, given that early parenthood concerns generally parents with low career opportunities, family allowances are justified on redistributive grounds. Children allowances differentiated according to the parent's age, would, if available, achieve that redistribution even better. Turning then to the optimal non-linear policy under asymmetric information, we showed that only children from parents with flat earnings profile should be subsidized, to solve the self-selection problem.

Finally, we investigated the impact of differential childbearing ages on the funding of social security, by introducing, in our set-up, a pension system. At the laissez-faire, one of the most striking effects of endogenous childbearing ages is that, under normal saving, or, alternatively, under a fully-funded pension system, individuals tend to adopt late childbearing with more earnings, whereas, under a PAYG system, there is a bias for early childbearing with overall lower earnings. We also showed, in a framework without heterogeneity, that, under a collective pension system, the fertility externalities related to early and late parenthoods differ significantly, inviting a larger Pigouvian subsidy for early children. The underlying reason is both qualitative (older children, under increasing productivity, can contribute more than younger, late children) and quantitative (early parents can, once retired, benefit from the contributions of their grandchildren). Turning back to the 2-type case, we then highlighted that the social planner faces, under endogenous childbearing age, a trade-off between efficiency (favoring late childbearing) and equity (helping early childbearing).

In sum, the present study, although it has no pretension to completeness, highlights some major challenges faced by policy-makers in the context of endogenous childbearing ages. Endogenous childbearing age affects many fundamental aspects of the optimal policy design. As far as redistribution is concerned, the observed correlation between the heterogeneity in terms of fertility age-pattern and in terms of career opportunities makes children allowances differentiated in terms of the age of parent socially desirable. Moreover, the internalization of fertility externalities in the context of pensions funding would also require children allowances differentiated according to the age of parents. Hence, even though policy debates are usually concerned with changes in the *total* fertility rate, the *timing* of births is far from a detail for the design of the optimal family policy, and, as such, will require more attention in the future.

References

[1] Balestrino, A., Cigno, A. & Pettini, A. (2002): Endogenous fertility and the design of family taxation, *International Tax and Public Finance*, 9, 175-193.

[2] Blackorby, C., Bossert, W. & Donaldson, D. (2005): *Population issues in social choice theory, welfare economics, and ethics*, Cambridge University Press, Cambridge.

[3] Cigno, A. (1986): Fertility and the tax-benefit system: a reconsideration of the theory of family taxation, *Economic Journal*, 96, 1035-1051.

[4] Cigno, A. (1991): *Economics of the family*, Clarendon Press, Oxford.

[5] Cigno, A. & Ermisch, J. (1989): A microeconomic analysis of the timing of births, *European Economic Review*, 33, 737-760.

[6] Cigno, A. & Luporini, A. (2009): Optimal family policy in the presence of moral hazard, when the quantity and the quality of children are stochastic, *IZA Discussion Paper* 4179.

[7] Cremer, H., Gahvari, F. & Pestieau, P. (2006): Pensions with endogenous and stochastic fertility, *Journal of Public Economics*, 90(12), 2303-2321.

[8] Cremer, H., Gahvari, F. & Pestieau, P. (2008): Pensions with heterogeneous individuals and endogenous fertility, *Journal of Population Economics*, 21(4), 961-981.

[9] D'Albis, H., Augeraud-Véron, E. & Schubert, K. (2010) Demographic-economic equilibria when the age at motherhood is endogenous. *Journal of Mathematical Economics*, 46(6), 1211-1221.

[10] Ermisch, J. (1988): The econometric analysis of birth rate dynamics in Britain. *The Journal of Human Resources*, 23(4), 563-576.

[11] Ermisch, J. & Ogawa, N. (1994): Age at motherhood in Japan. *Journal of Population Economics*, 7, 393-420.

[12] Ermisch, J. & Pevalin, D. (2005): Early motherhood and later partnerships. *Journal of Population Economics*, 18, 469-489.

[13] Gahvari, F. (2009): Pensions and fertility: in search of a link, *International Tax and Public Finance*, 16, 418-442.

[14] Gauthier, A. (2007): The impact of family policies on fertility in industrialized countries: a review of the literature, *Population Research and Policy Review*, 26, 323-346.

[15] Gilbert, W., Nesbitt, T. & Danielsen, B. (1999): Childbearing beyond age 40: pregnancy outcome in 24032 cases, *Obstetrics and Gynaecology*, 93, 9-14.

[16] Gustafsson, SS. (2001): Optimal age at motherhood. Theoretical and empirical considerations on postponement of maternity in Europe, *Journal of Population Economics*, 14, 225-247.

[17] Happel, S.K., Hill, J.K. & Low S.A. (1984): An economic analysis of the timing of childbirth, *Population Studies*, 38, 299-311.

[18] Heckman, J. & Walker, J. (1990): The relationship between wages and income and the timing and spacing of births: evidence from Swedish longitudinal data, *Econometrica*, 58, 1411-1441.

[19] INED (2011): Evolution de la fertilité. Data available online at http://www.ined.fr/fr/pop_chiffres/france/naissances_fécondite/evolution_fecondite/

[20] Joshi, H. (1990): The cash alternative costs of childbearing: an approach to estimation using British data, *Population Studies*, 44, 41-60.

[21] Joshi, H. (1998): The opportunity costs of childbearing: more than mother's business, *Journal of Population Economics*, 11, 161-183.

[22] Joshi, H. (2002): Production, reproduction, and education: women, children and work in a British perspective. *Population and Development Review*, 28, pp. 445-474.

[23] Laroque, G. & Salanie, B. (2004): Fertility and financial incentives in France, *CESifo Economic Studies*, 50(3), 423-450.

[24] Merrigan, P. & St-Pierre, Y. (1998): An econometric and neoclassical analysis of the timing and spacing of births in Canada from 1950 to 1990, *Journal of Population Economics*, 11, 29-51.

[25] Pestieau, P. & Ponthiere, G. (2011): Optimal fertility along the lifecycle, *CORE Discussion Paper* 2011-33.

[26] Samuelson, P. (1975): The optimum growth rate for population, *International Economic Review*, 16(3), 531-538.

[27] Schultz, T.P. (1985): Changing world prices, women's wages and the fertility transition: Sweden 1860-1910, *Journal of Political Economy*, 93, 1126-1154.

[28] Tasiran, A.C. (1995): *Fertility dynamics. Spacing and timing of births in Sweden and the United States*, Elsevier, Amsterdam.

[29] Walker, J.R. (1995): The effect of public policies on recent Swedish fertility behaviour, *Journal of Population Economics*, 8, 223-251.